Kinematics & Dynamics of Linkages

Lecture 6: Planetary Gears

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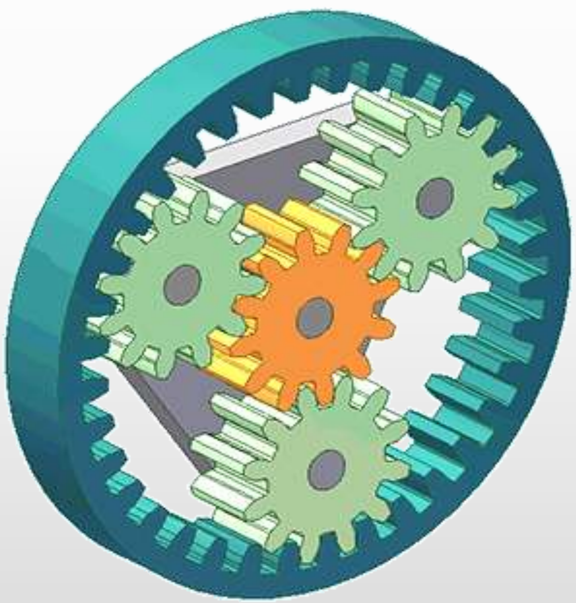
Planetary Gear Trains

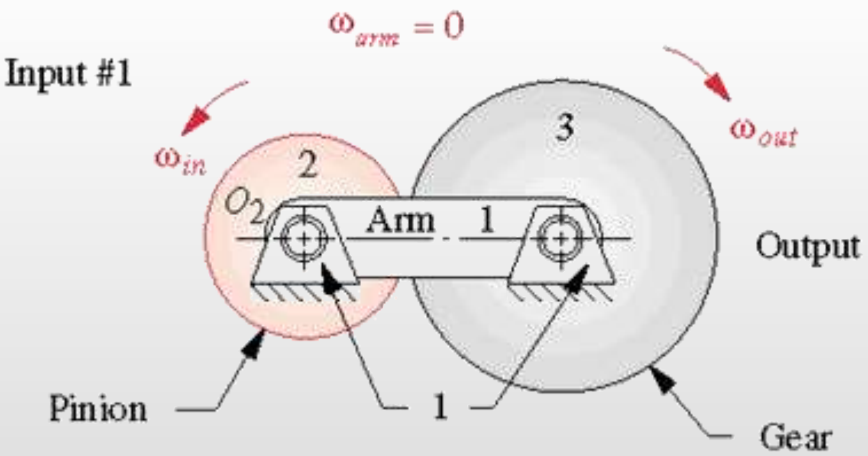
Planetary gearset: A **planet** gear orbits around a **sun** gear



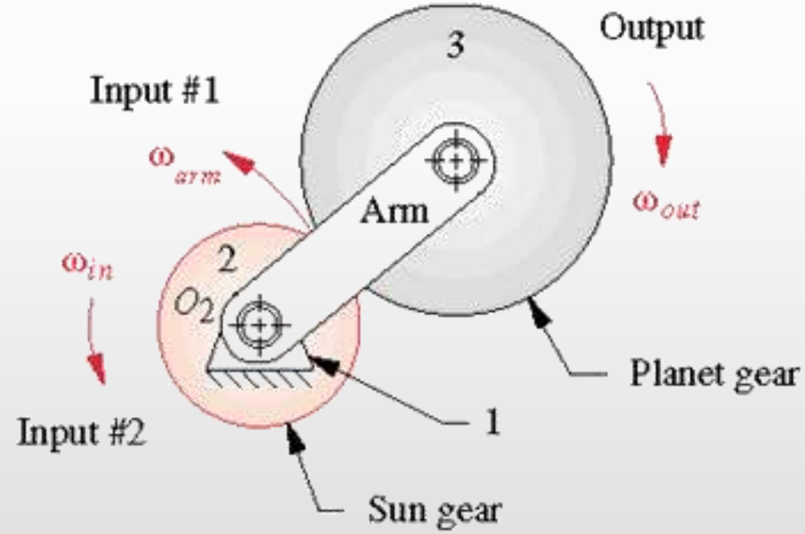
http://bestanimations.com/Science/Gears/Gears2.html

Planetary Gear Trains

- A planetary set is a 2DOF system requiring 2 inputs (arm and sun)
- If either is held, the set is 1DOF which is the conventional case



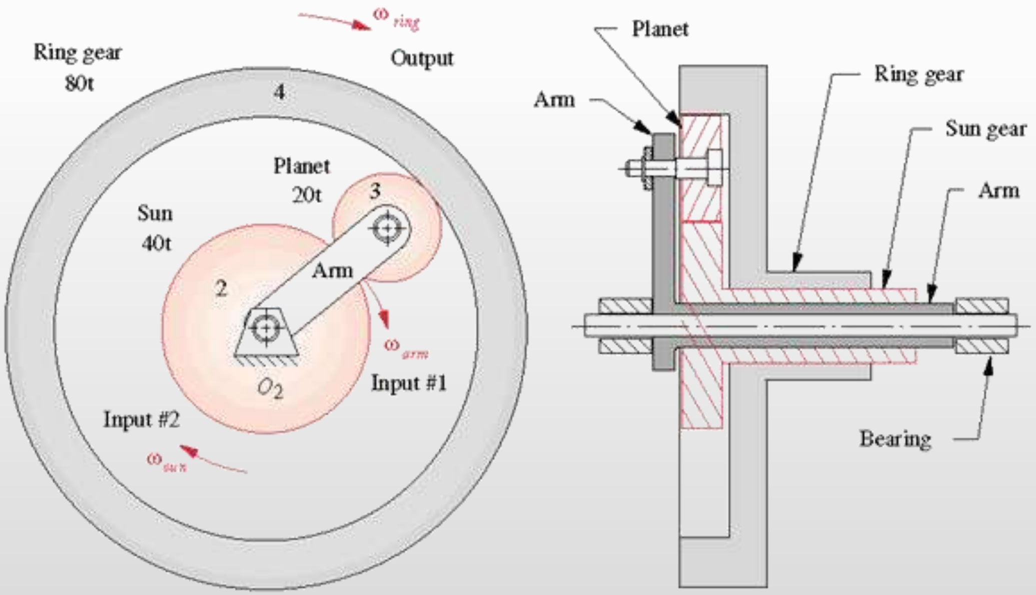
Conventional



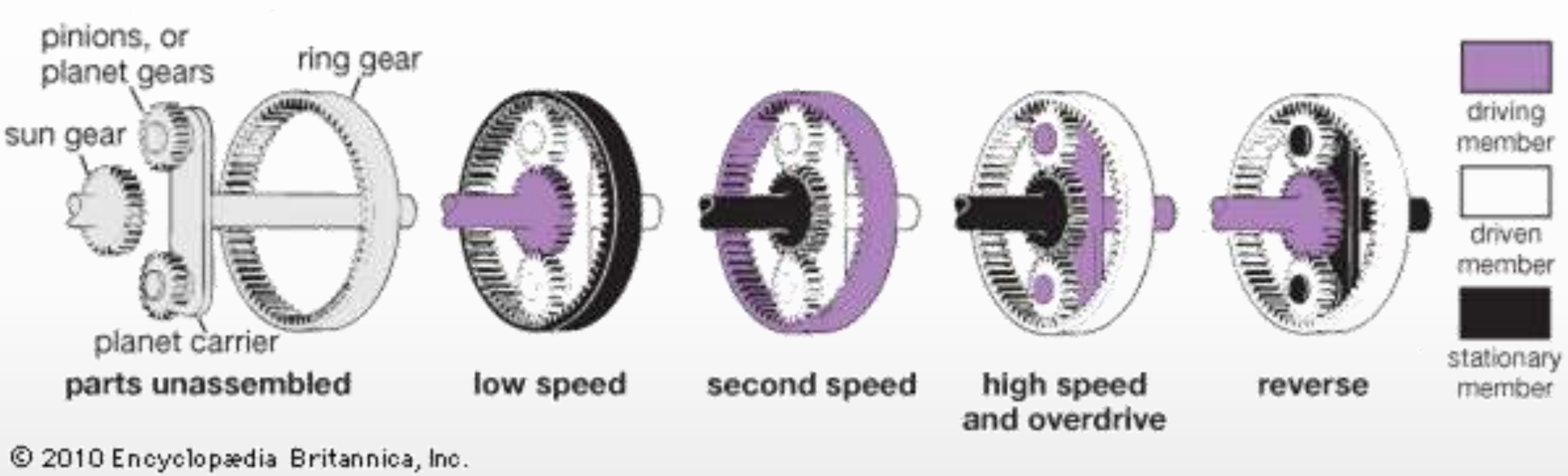
Planetary

Useful planetary configuration

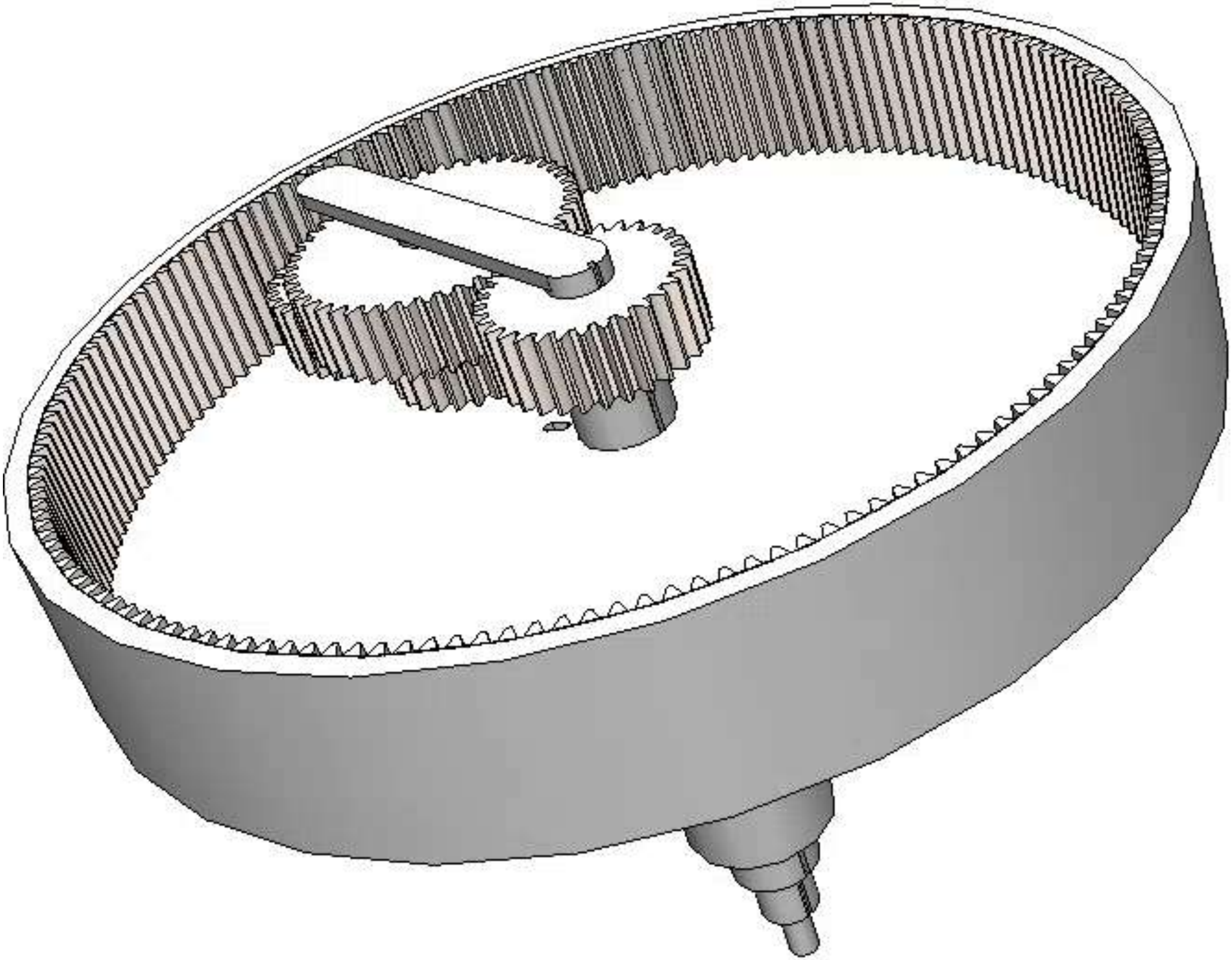
- A more useful configuration is where a ring gear is added
- This will bring the gear output back to a grounded pivot (O_2)



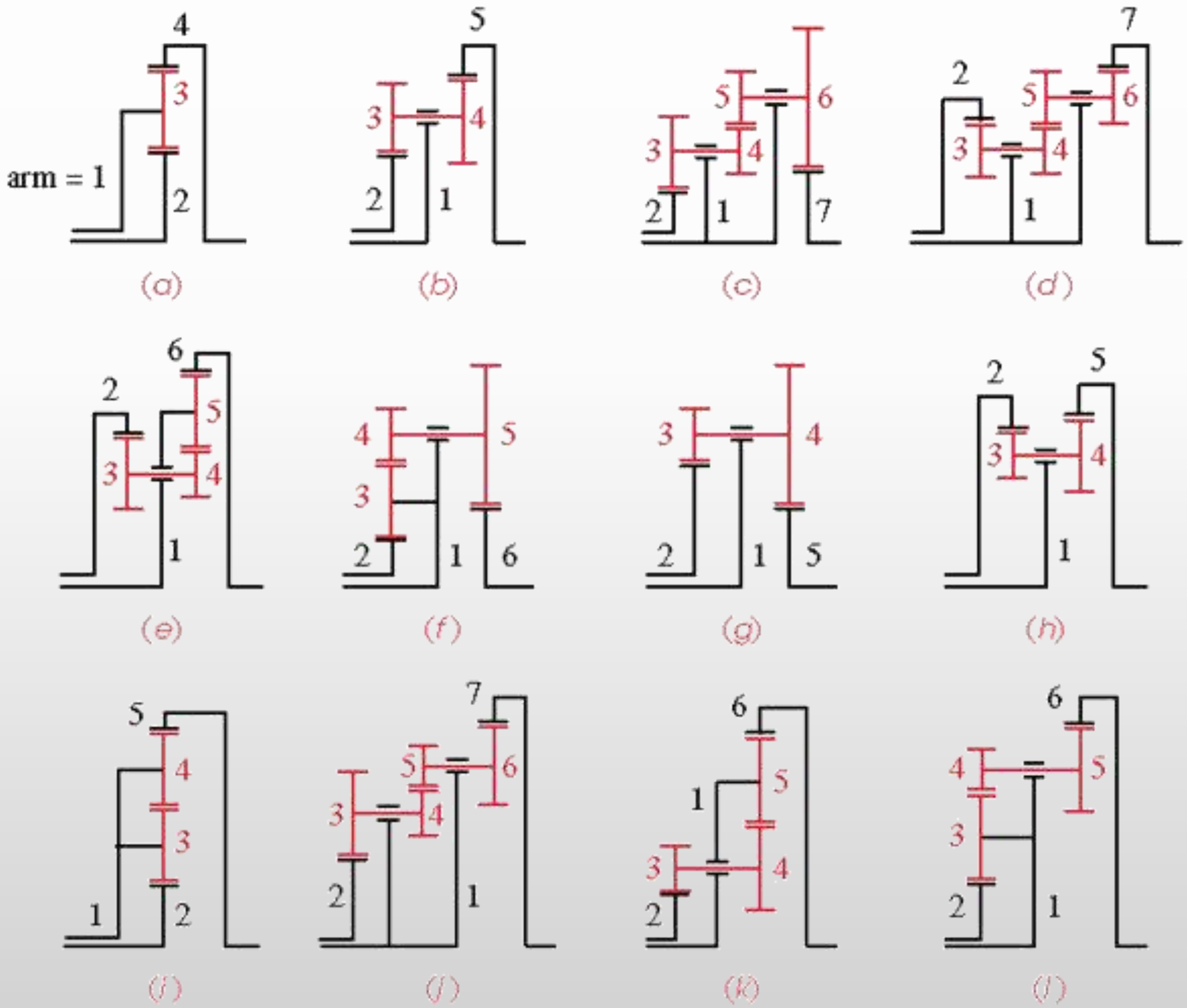
Planetary transmission



https://media1.britannica.com/eb-media/43/104143-004-D11FFB27.gif



Levai’s planetary gearsets configurations



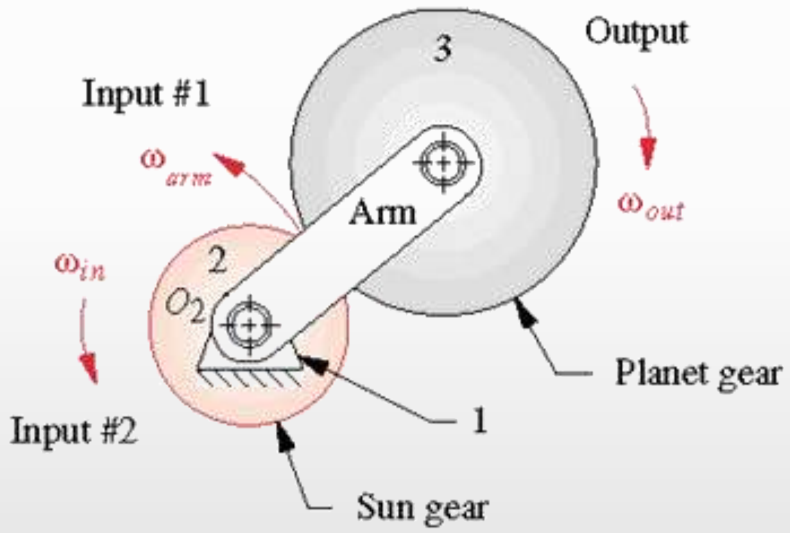
Planetary gearsets equations

$$\omega_{gear} = \omega_{arm} + \omega_{gear/arm}$$

$$m_v = \frac{\omega_{out}}{\omega_{in}} = \pm\frac{d_{in}}{d_{out}} = \pm\frac{N_{in}}{N_{out}}$$

$$\frac{\omega_{gear1}}{\omega_{gear2}} = \pm\frac{N_{gear2}}{N_{gear1}}$$

$$\frac{\omega_{gear1/arm}}{\omega_{gear2/arm}} = \pm\frac{N_{gear2}}{N_{gear1}}$$

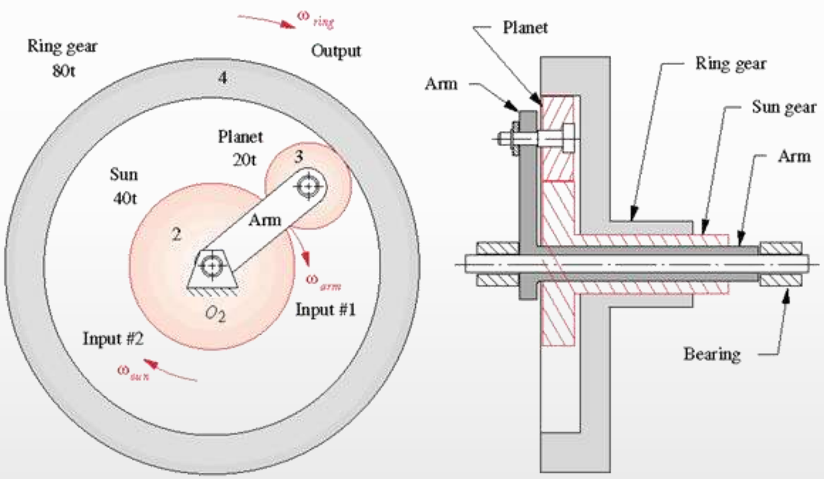


Example 1

Find the absolute output angular velocity of the ring gear

Sun gear	$N_2 = 40$
Planet gear	$N_3 = 20$
Ring gear	$N_4 = 80$

Input to arm	200 rpm cw
Input to sun	100 rpm cw



Example 1 Solution – Tabular method

Start by finding the missing values in the table below

Gear #	ω_{gear} =	ω_{arm} +	$\omega_{gear/arm}$
2	-100	-200	a
3	d	-200	b
4	e	-200	c

Gear ratio
$-40/20$
$+20/80$

Example 1 Solution – Tabular method

Solving for a, b, c, d, and e we get:

$$a = -100 + 200 = 100$$

$$\frac{b}{a} = -\frac{40}{20} \rightarrow b = -a\frac{40}{20} = -200$$

$$\frac{c}{b} = +\frac{20}{80} \rightarrow c = +b\frac{20}{80} = -50$$

$$d = -200 - 200 = -400$$

$$e = -200 - 50 = -250$$

Gear #	ω_{gear} =	ω_{arm} +	$\omega_{gear/arm}$	Gear ratio
2	−100	−200	a	
3	d	−200	b	−40/20
4	e	−200	c	+20/80

Gear #	ω_{gear} =	ω_{arm} +	$\omega_{gear/arm}$	Gear ratio
2	−100	−200	+100	
3	−400	−200	−200	−40/20
4	−250	−200	−50	+20/80

The absolute output angular velocity of the ring gear is **250 cw**

Example 1 Solution – Formula method

Similar to compound gears but with relative velocities.

ω_F the angular velocity of the first gear

ω_L the angular velocity of the last gear

For the first gear $\quad \omega_{F/arm} = \omega_F - \omega_{arm}$

For the last gear $\quad \omega_{L/arm} = \omega_L - \omega_{arm}$

$$\frac{\omega_{out}}{\omega_{in}} = \frac{\omega_{L/arm}}{\omega_{F/arm}} = \frac{\omega_L - \omega_{arm}}{\omega_F - \omega_{arm}} = \pm \frac{product\ of\ Number\ of\ teeth\ of\ driver\ gears}{product\ of\ Number\ of\ teeth\ of\ driven\ gears}$$

Both first and last gear must be pivoted to ground, and there must be a path of meshes connecting them.

Example 1 Solution – Formula method

$$\omega_F = \omega_2 = -100$$

$$\omega_L = \omega_4 \ \text{ to be found}$$

For the first gear $\quad \omega_{F/arm} = \omega_F - \omega_{arm} = -100 - (-200) = 100$

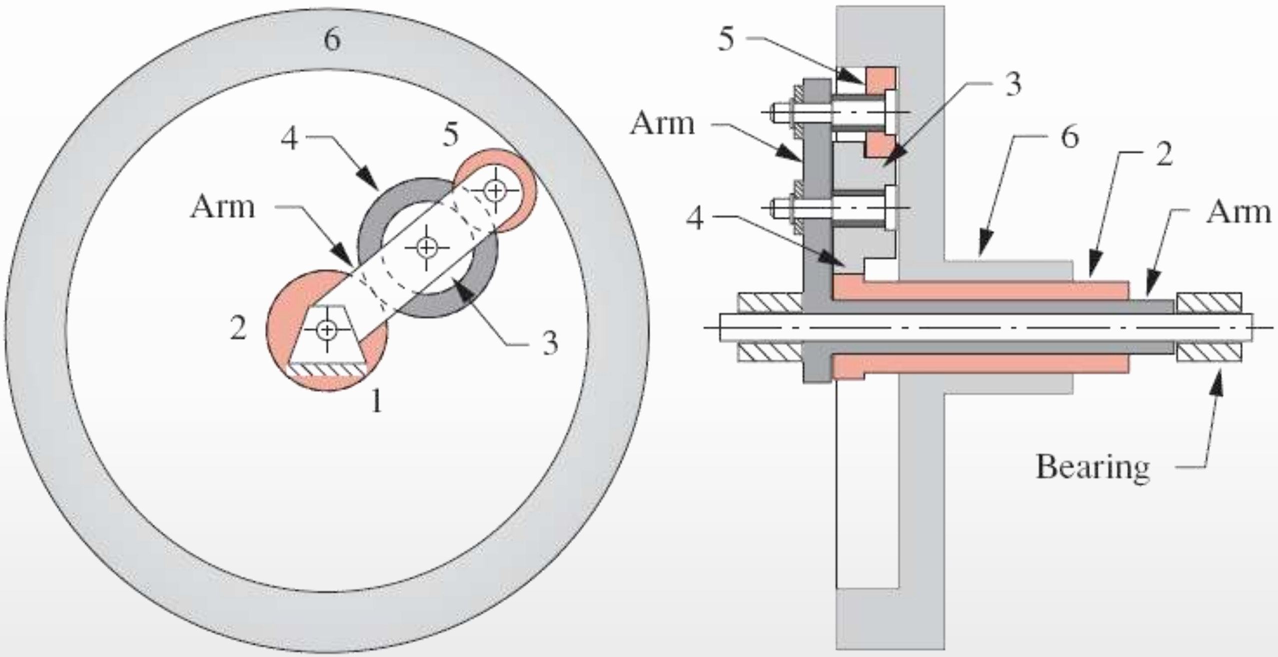
For the last gear $\quad \omega_{L/arm} = \omega_L - \omega_{arm} = \omega_4 - (-200) = \omega_4 + 200$

$$\frac{\omega_{L/arm}}{\omega_{F/arm}} = \frac{\omega_L - \omega_{arm}}{\omega_F - \omega_{arm}} = \left(-\frac{N_2}{N_3}\right)\left(+\frac{N_3}{N_4}\right) = \left(-\frac{40}{20}\right)\left(+\frac{20}{80}\right) = -\frac{1}{2}$$

$$\frac{\omega_L - \omega_{arm}}{\omega_F - \omega_{arm}} = \frac{\omega_4 + 200}{100} = -\frac{1}{2} \quad \rightarrow \quad \omega_4 = -250$$

The absolute output angular velocity of the ring gear is **250 cw**

Example 2



$$N_2 = 30 \quad N_3 = 25 \quad N_4 = 45 \quad N_5 = 30 \quad N_6 = 160$$

$$\omega_6 = 40 \quad \omega_{arm} = -50 \quad \textit{Find } \omega_2 \textit{ and } \omega_3$$

Example 2 Solution

$\omega_F = \omega_2$ to be found

$\omega_L = \omega_6 = 40$

For the first gear $\omega_{F/arm} = \omega_F - \omega_{arm} = \omega_2 - (-50) = \omega_2 + 50$

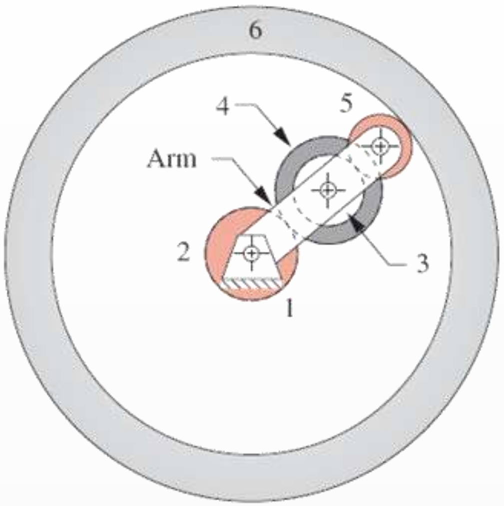
For the last gear $\omega_{L/arm} = \omega_L - \omega_{arm} = 40 - (-50) = 90$

$$\frac{\omega_{L/arm}}{\omega_{F/arm}} = \frac{\omega_L - \omega_{arm}}{\omega_F - \omega_{arm}} = \left(-\frac{N_2}{N_4}\right)\left(-\frac{N_3}{N_5}\right)\left(\frac{N_5}{N_6}\right) = \left(-\frac{30}{45}\right)\left(-\frac{25}{30}\right)\left(\frac{30}{160}\right) = \frac{5}{48}$$

$$\frac{\omega_L - \omega_{arm}}{\omega_F - \omega_{arm}} = \frac{90}{\omega_2 + 50} = \frac{5}{48} \quad \rightarrow \quad \omega_2 = 814$$

The absolute output angular velocity of the first gear is **814 ccw**

Example 2 Solution



$$\omega_3 = \omega_4 \text{ on the same shaft}$$

$$\frac{\omega_4 - \omega_{arm}}{\omega_2 - \omega_{arm}} = \frac{\omega_3 - \omega_{arm}}{\omega_2 - \omega_{arm}} = -\frac{N_2}{N_4} = -\frac{30}{45}$$

$$\omega_3 = -\frac{N_2}{N_4}\left(\omega_2 - \omega_{arm}\right) + \omega_{arm} = -\frac{30}{45}\left(814 + 50\right) - 50 \quad \rightarrow \quad \omega_3 = -626$$

The absolute output angular velocity of the first gear is **626 cw**

SolidWorks Simulation